

Finite-time response function of uniformly accelerated entangled atoms

C. D. Rodríguez-Camargo¹, G. Menezes^{1,2} and N. F. Svaiter¹

¹ Centro Brasileiro de Pesquisas Físicas, 22290-180 Rio de Janeiro, RJ, Brazil

² Grupo de Física Teórica e Matemática Física, Departamento de Física,
Universidade Federal Rural do Rio de Janeiro, 23897-000 Seropédica, RJ, Brazil

E-mail: christian@cbpf.br, gabrielmenezes@ufrrj.br and nfuxsvai@cbpf.br

Abstract. We examine the entanglement generation between uniformly accelerated two-level atoms weakly coupled with a massless scalar field in Minkowski vacuum. Using time-dependent perturbation theory we evaluate a finite-time response function and we identify the mutual influence of atoms via the quantum field as a coherence agent in each response function terms. The associated thermal spectrum perceived by the atoms is found for a long observational time interval. In addition, we study the mean life of entangled states for different accelerations. The possible relevance of our results is discussed.

PACS numbers: 03.65.Ud, 03.67.-a, 04.62.+v, 42.50.Lc, 11.10.-z

1. Introduction

A detector moving with constant proper acceleration a perceives the Minkowski vacuum state of a quantum field as a thermal bath with temperature [1, 2]

$$T = \frac{\hbar a}{2\pi c k_B},$$

where \hbar, k_B, c are the Planck and Boltzmann's constants and the speed of light, respectively. Using perturbation theory in first-order approximation it can be shown that the transition rates of the Unruh-DeWitt detector [3] interacting with a scalar field in the Minkowski vacuum is given by the Fourier transform of the positive frequency Wightman function evaluated on the world line of the detector [4]. In the case of an uniformly accelerated detector, it is found that, if it is initially prepared in its ground state, it can be excited by the thermal radiation perceived by it [5].

Quantum entanglement is considered as one of the key features of quantum information processing. Several sources of entangled quantum systems have been discussed in literature, for instance, in solid-state physics, quantum optics, and also atoms in cavity quantum electrodynamics [6]. Some examples of generation of entangled systems of two-level atoms interacting with a bosonic field can be found in Refs. [7, 8].

Aside from production of those entangled systems, quantum-information processing requires the presence of a strong coherent coupling between the entities of the system. Nevertheless, under realistic experimental conditions, entanglement is degraded through uncontrolled coupling to environment [9].

In recent years the field of relativistic quantum information has emerged as an important research topic and is generating increasing interest within the scientific community. The mutual influence of atoms through their interaction with quantum fields is an important stimulating issue in order to analyse decoherence properties [10–12]. Some works studying quantum entanglement in different setups are given by Refs. [12–18]. In turn, for a wide set of results in this area and special issues of performing satellite experiments, we refer the reader the Ref. [19]. Many of such works demonstrate that entanglement is an observer-dependent quantity. In addition, recent works point towards the importance of considering explicitly the contributions of vacuum fluctuations and radiation reaction in the radiative processes of entangled atoms [20,21]. Within such a context, we also refer the reader the Refs. [22,23].

In the present work we wish to address different issues in comparison with the aforementioned papers. Here we are interested in studying the entanglement generation between uniformly accelerated two-level atoms due to the vacuum fluctuations of a quantum scalar field. We remark that this simplified situation contains all the important ingredients to study quantum entanglement between atoms. We propose to give a thorough understanding on how the distance between the atoms can be employed as a control parameter for entanglement generation from the vacuum state. On the other hand, an important issue that naturally arises in this context is whether these recently formed entangled states could persist for long time intervals. In order to pursue an answer to this question, one may investigate the mean life of the entangled states, which is one of the topics of the present work.

The aim of this paper is to study radiative processes of a pair of uniformly accelerated two-level atoms interacting with a quantum massless scalar field. We evaluate the transition rates within time-dependent perturbation theory in a finite time interval. Refs. [24–26] present investigations on the excitation probability evaluated in a finite time interval for Unruh-DeWitt detectors. The paper is organized as follows. In Sec. II we discuss the Hamiltonian of two identical two-level atoms weakly coupled with a massless scalar field in Minkowski vacuum. In Sec. III we evaluate the associated response functions. We mainly focus our attention in the so-called crossed response functions and we compute the general expression for different accelerations and time intervals for the symmetric state transition. We present the total transition rate for equal accelerations of the atoms. In Sec. IV we study the mean life of the symmetric entangled state. In Sec. V we present a discussion of the obtained results. Henceforth we work with units such that $\hbar = c = k_B = 1$ and the Minkowski metric we use is given by $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$. We denote the coordinate time by $x^0 = t$.

2. Two identical atoms coupled with a massless scalar field

Let us consider two identical two-level atoms interacting with a massless scalar field in a four-dimensional Minkowski space-time. Here we consider that the atoms are moving along different hyperbolic trajectories. Let us first establish the dependence between the proper times by the structure of Rindler coordinates [27]

$$t = a^{-1}e^{a\xi} \sinh(a\eta), \quad z = a^{-1}e^{a\xi} \cosh(a\eta). \quad (1)$$

The lines of constant η are straight lines ($z \sim t$), whereas the lines of constant ξ are hyperbolae $z^2 - t^2 = a^{-2}e^{2a\xi} \equiv \text{constant}$. As such, they represent the world lines of uniformly accelerated observers in Minkowski space-time. The proper acceleration is defined by $ae^{-a\xi} = \alpha^{-1}$, and the proper time of the atoms τ is related to ξ and η by $\tau = e^{a\xi}\eta$. We assume that the j -th atom accelerates uniformly with acceleration α_j^{-1} , $j = 1, 2$. The dependence between the proper times is given by the lines of constant η , such that

$$\tau_2(\tau_1) = \tau_1 e^{a(\xi_2 - \xi_1)}, \quad (2)$$

with $e^{a(\xi_2 - \xi_1)} = \alpha_2/\alpha_1$.

The total Hamiltonian of the system with respect to the coordinate time t can be written as

$$H = H_A + H_F + H_{int}, \quad (3)$$

where H_A is the free atomic Hamiltonian, H_F is the free field Hamiltonian and H_{int} describes the interaction between the atoms and the fields. Let us briefly discuss each of such terms. We may express the atomic Hamiltonian in the Dicke notation as [28]

$$H_A = \frac{\omega_0}{2} \left[S_1^z(\tau) \otimes \mathbb{1}_2 \frac{d\tau_1}{dt} + \mathbb{1}_1 \otimes S_2^z(\tau) \frac{d\tau_2}{dt} \right] \quad (4)$$

where $S_i^z = (|e_i\rangle\langle e_i| - |g_i\rangle\langle g_i|)/2$ is associated with the i -th atom and $|g_i\rangle$, $|e_i\rangle$ is the ground and excited state of the i -th atom, respectively. The eigenstates and respective energies are given by

$$\begin{aligned} E_{gg} &= -\omega_0 & |gg\rangle &= |g_1\rangle |g_2\rangle, \\ E_{ge} &= 0 & |ge\rangle &= |g_1\rangle |e_2\rangle, \\ E_{eg} &= 0 & |eg\rangle &= |e_1\rangle |g_2\rangle, \\ E_{ee} &= \omega_0 & |ee\rangle &= |e_1\rangle |e_2\rangle, \end{aligned} \quad (5)$$

where a tensor product is implicit. Another possible choice is the Bell-state basis. The Bell states are known as the four maximally entangled two-qubit states. In terms of the above product states, the Bell states are expressed as

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}} (|g_1\rangle |e_2\rangle \pm |e_1\rangle |g_2\rangle), \quad (6)$$

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}} (|g_1\rangle |g_2\rangle \pm |e_1\rangle |e_2\rangle). \quad (7)$$

The Hamiltonian (4) is showing a degeneracy associated with the eigenstates $|ge\rangle$ and $|eg\rangle$. Any linear combination of these degenerate eigenstates will be an eigenstate of the atomic Hamiltonian and these linear combination must have the same degenerate energy value. Then (6) are eigenstates of H_A . On the other hand, it is clear that (7) are not eigenstates of the atomic Hamiltonian.

The free Hamiltonian of the quantum field which governs the field time evolution is given by

$$H_F = \frac{1}{2} \int d^3x [(\dot{\varphi}(x))^2 + (\nabla\varphi(x))^2], \quad (8)$$

where the dot represents the derivative with respect to t .

At this point we remark that we will be working in the interaction representation. Hence we assume that the coupling between the atoms and the field is described by a monopole interaction in the form

$$H_{int}(t) = \sum_{j=1}^2 \mu_j m^{(j)}(\tau_j(t)) \varphi[x_j(\tau_j(t))] \frac{d\tau_j(t)}{dt}, \quad (9)$$

where in the interaction picture we have

$$m^{(k)}(\tau_j) = e^{iH_A\tau_j} m^{(k)}(0) e^{-iH_A\tau_j}, \quad (10)$$

where

$$m^{(j)}(0) = |e^{(j)}\rangle\langle g^{(j)}| + |g^{(j)}\rangle\langle e^{(j)}| \quad (11)$$

is the monopole moment operator of the j -th atom. In addition, the quantity μ_j is the coupling constant of the j -th atom, $\varphi[x_j(\tau_j(t))]$ is the field at the point of the j -th atom. Henceforth we set $\mu_1 = \mu_2 = \mu$ and we assume that μ is small.

As mentioned above, we consider that the atoms are moving along world lines $x_{1,2}^\nu(\tau_{1,2})$ parametrized by the proper times $\tau_{1,2}$. Since in this case the proper times of the atoms do not coincide, we write the time-evolution operator as [17]

$$U = \exp \left\{ -i \int d\tau_1 \mu \left[m^{(1)}(\tau_1) \varphi(x_1^\nu(\tau_1)) + m^{(2)}(\tau_2(\tau_1)) \varphi(x_2^\nu(\tau_2(\tau_1))) \frac{d\tau_2(\tau_1)}{d\tau_1} \right] \right\}. \quad (12)$$

Preparing the field φ in the Minkowski vacuum state $|0_M\rangle$ and the atoms in the collective ground state $|gg\rangle$, the amplitude in first-order perturbation theory for a transition to the final state $|\omega; \Omega\rangle$ is given by

$$\begin{aligned} A_{|gg;0_M\rangle \rightarrow |\omega;\Omega\rangle} &= i\mu \left\langle \omega; \Omega \right| \int_{\tau_0}^{\tau_f} d\tau_1 \left[m^{(1)}(\tau_1) \varphi(x_1^\nu(\tau_1)) \right. \\ &\quad \left. + m^{(2)}(\tau_2(\tau_1)) \varphi(x_2^\nu(\tau_2(\tau_1))) \frac{d\tau_2(\tau_1)}{d\tau_1} \right] \left| gg; 0_M \right\rangle. \end{aligned} \quad (13)$$

The transition probability to all possible $|\omega; \Omega\rangle$ up to order μ^2 is obtained by squaring the modulus of the above expression and summing over ω and the complete set Ω .

Hereafter we will be interested solely in the transition probability associated with a particular transition $|\omega'\rangle \rightarrow |\omega\rangle$. In first-order approximation this is given by

$$\Gamma_{|\omega'\rangle \rightarrow |\omega\rangle}(\Delta\omega, \tau_0, \tau_f) = \mu^2 \sum_{i,j} \left[m_{\omega\omega'}^{(i)*} m_{\omega\omega'}^{(j)} F_{ij}(\Delta\omega, \tau_0, \tau_f) \right] \quad (14)$$

where $\Delta\omega = \omega - \omega'$, $i = 1, 2$, $j = 1, 2$ and the matrix elements are given by

$$\begin{aligned} m_{\omega\omega'}^{(1)} &= \langle \omega | m^{(1)} \otimes \mathbb{1}_2 | \omega' \rangle \\ m_{\omega\omega'}^{(2)} &= \langle \omega | \mathbb{1}_1 \otimes m^{(2)} | \omega' \rangle. \end{aligned}$$

Note that ω can be any of the energies given in Eq. (5) and also $|\omega\rangle$ can be any of the states $\{|gg\rangle, |\Psi^\pm\rangle, |ee\rangle\}$. The corresponding response functions are defined by

$$\begin{aligned} F_{ij}(\Delta\omega, \tau_0, \tau_f) &= \int_{\tau_0}^{\tau_f} d\tau_1 \int_{\tau_0}^{\tau_f} d\tau'_1 e^{-i\Delta\omega(\tau_i(\tau_1) - \tau_j(\tau'_1))} \\ &\times G_{ij}^+(\tau_1, \tau'_1) \frac{d\tau_j(\tau'_1)}{d\tau'_1} \frac{d\tau_i(\tau_1)}{d\tau_1}, \end{aligned} \quad (15)$$

where $G_{ij}^+(\tau_1, \tau'_1) = \langle 0_M | \varphi(x_i(\tau_i(\tau_1))) \varphi(x_j(\tau_j(\tau'_1))) | 0_M \rangle$ is the positive-frequency Wightman function in Minkowski space-time for a massless scalar field, which is given by

$$G_{ij}^+(\tau, \tau') = \frac{1}{8\pi^2} \frac{1}{\sigma(\tau, \tau')}, \quad (16)$$

where $\sigma(\tau, \tau')$ is given by

$$\begin{aligned} 2\sigma(\tau, \tau') &= (x_i(\tau) - x_j(\tau'))^2 \\ &= -(t_i(\tau) - t_j(\tau') - i\epsilon)^2 + |\mathbf{x}_i(\tau) - \mathbf{x}_j(\tau')|^2. \end{aligned}$$

We are interested in the entanglement generation of a pair of atoms travelling in different hyperbolic world lines. Hence we study the transition $|gg\rangle \rightarrow |\Psi^\pm\rangle$, with $\Delta\omega = \omega_0 > 0$. The appearance of cross terms in the transition probability has its origin in the fact of working with two atoms, both interacting with a common scalar quantum field.

We may define the associated transition rate as follows

$$\mathcal{R}_{|\omega'\rangle \rightarrow |\omega\rangle}(\Delta\omega, \Delta t) = \frac{d\Gamma_{|\omega'\rangle \rightarrow |\omega\rangle}(\Delta\omega, \Delta t)}{d(\Delta t)}, \quad (17)$$

where $\Gamma_{|\omega'\rangle \rightarrow |\omega\rangle}(\Delta\omega, \Delta t)$ is given by Eq. (14). In order to present an explicit expression for the transition rate, one needs to properly evaluate in detail each of the response functions $F_{ij}(\Delta\omega, \tau_0, \tau_f)$. This is the topic of the next Section.

3. Discussion on the response functions

3.1. Individual response functions, $F_{11}(\Delta\omega, \Delta t)$ and $F_{22}(\Delta\omega, \Delta t)$

Let us first evaluate the contribution $F_{11}(\Delta\omega)$ to the total response function. Inserting Eq. (1) in expression (16) one finds all the relevant positive-frequency Wightman functions of the problem. In particular, the Wightman function G_{11}^+ is given by

$$G_{11}^+(\tau_1 - \tau'_1) = -\frac{1}{16\pi^2 \alpha_1^2 \sinh^2\left(\frac{\tau_1 - \tau'_1}{2\alpha_1} - \frac{i\epsilon}{\alpha_1}\right)}. \quad (18)$$

Using known series identities [29] we can rewrite (18) as

$$G_{11}^+(\tau_1 - \tau'_1) = -\frac{1}{4\pi^2} \sum_{n=-\infty}^{\infty} ((\tau_1 - \tau'_1) - 2i\epsilon + 2\pi i\alpha_1 n)^{-2}. \quad (19)$$

Changing variables to

$$\psi = \tau_1 - \tau'_1 \quad \eta = \tau_1 + \tau'_1, \quad (20)$$

we have,

$$F_{11}(\Delta\omega, \Delta t) = \frac{1}{2} \int_{-\Delta t}^{\Delta t} d\psi (2|\psi| - 2\Delta t) e^{-i\Delta\omega\psi} G_{11}^+(\psi), \quad (21)$$

where $\Delta t = \tau_f - \tau_0$. The evaluation of the integral leads us to [24]

$$\begin{aligned} F_{11}(\Delta\omega, \Delta t) = & \frac{\Delta t}{2\pi^2} \left\{ \pi|\Delta\omega|\Theta(-\Delta\omega) + |\Delta\omega| \left(\text{Si}(\Delta\omega\Delta t) - \frac{\pi}{2} \right) \right. \\ & + \frac{\pi|\Delta\omega|}{e^{2\pi\alpha_1|\Delta\omega|} - 1} + \int_{\Delta t}^{\infty} d\psi \cos(\Delta\omega\psi) \left[\frac{1/(2\alpha_1)^2}{\sinh^2[\psi/(2\alpha_1)]} - \frac{1}{\psi^2} \right] \Big\} \\ & + \frac{1}{2\pi^2} \left\{ \cos(\Delta\omega\Delta t) + \log\left(\frac{\Delta t}{2\pi\epsilon}\right) - 1 \right. \\ & + \int_0^{\Delta t} d\psi \frac{\cos(\Delta\omega\psi) - 1}{\psi} \\ & + \left. \int_0^{\Delta t} d\psi \psi \cos \Delta\omega\psi \left[\frac{1/(2\alpha_1)^2}{\sinh^2[\psi/(2\alpha_1)]} - \frac{1}{\psi^2} \right] \right\}. \end{aligned} \quad (22)$$

For details concerning such a calculation we refer the reader the Appendix A. We are interested in the rate

$$R_{ij}(\Delta\omega, \Delta t) = \frac{dF_{ij}(\Delta\omega, \Delta t)}{d(\Delta t)} \quad (23)$$

which is related to the mean life of states. From the expression (A.3) found in Appendix A we obtain that for large time intervals we have the following expression

$$\begin{aligned} \lim_{\Delta t \rightarrow \infty} R_{11}(\Delta\omega, \Delta t) = & \frac{|\Delta\omega|}{2\pi} \left\{ \Theta(-\Delta\omega) \left[1 + \frac{1}{e^{2\pi\alpha_1|\Delta\omega|} - 1} \right] \right. \\ & + \left. \Theta(\Delta\omega) \frac{1}{e^{2\pi\alpha_1\Delta\omega} - 1} \right\}. \end{aligned} \quad (24)$$

The above equation shows us that the equilibrium between the uniformly accelerated atom and scalar field in the Minkowski vacuum state $|0_M\rangle$ is the same as that which would have been achieved had this atom followed an inertial trajectory but immersed in a bath of thermal radiation at the temperature $\beta_1^{-1} = 1/2\pi\alpha_1$.

Analogously, for the response function F_{22} ,

$$G_{22}^+(\tau_2(\tau_1), \tau_2(\tau'_1)) = -\frac{1}{16\pi^2\alpha_2^2 \sinh^2\left(\frac{\tau_1 - \tau'_1}{2\alpha_1} - \frac{i\epsilon}{\alpha_1}\right)}. \quad (25)$$

Performing similar steps as before, we can rewrite (25) as

$$G_{22}^+(\psi) = -\frac{1}{4\pi^2} \frac{\alpha_1^2}{\alpha_2^2} \sum_{n=-\infty}^{\infty} (\psi - 2i\epsilon + 2\pi i\alpha_1 n)^{-2}, \quad (26)$$

such that, the expression for the response function $F_{22}(\Delta\omega, \Delta t)$ yields

$$F_{22}(\Delta\omega, \Delta t) = -\frac{1}{2}e^{2a(\xi_2-\xi_1)} \int_{-\Delta t}^{\Delta t} d\psi G_{22}^+(\psi) \times (-2|\psi| + 2\Delta t)e^{-i\Delta\omega e^{a(\xi_2-\xi_1)}\psi}. \quad (27)$$

From the expression (A.6) in the Appendix A, we obtain an asymptotic expression $\Delta t \rightarrow \infty$ for $R_{22}(\Delta\omega, \Delta t)$ which is just Eq. (24) with the replacement $\alpha_1 \rightarrow \alpha_2$. We observe that, for large time intervals, the equilibrium between the atom 2 and scalar field in the Minkowski vacuum state is the same as the equilibrium of this atom in an inertial trajectory and a bath of thermal radiation at the temperature $\beta_2^{-1} = 1/2\pi\alpha_2$.

3.2. Crossed response functions, $F_{12}(\Delta\omega, \Delta t)$ and $F_{21}(\Delta\omega, \Delta t)$

The first two response functions discussed above correspond to individual atomic transitions. Therefore one expects that the response functions $F_{12}(\Delta\omega, \Delta t)$ and $F_{21}(\Delta\omega, \Delta t)$ exhibit the existence of cross-correlations between the atoms mediated by the field. In order to unveil such a behavior, we now proceed to evaluate such contributions. It is easy to show that the positive frequency Wightman functions for both cases are equal and are given by

$$G_{21}^+(\tau_2(\tau_1), \tau_1') = G_{12}^+(\tau_1, \tau_2(\tau_1')) = G_c^+(\tau_1 - \tau_1') \quad (28)$$

where

$$G_c^+(\tau_1 - \tau_1') = -\frac{1}{16\pi^2\alpha_1\alpha_2}G_{c_0}^+(\tau_1 - \tau_1') \quad (29)$$

and

$$G_{c_0}^+(\tau_1 - \tau_1') = \left[\sinh\left(\frac{\tau_1 - \tau_1'}{2\alpha_1} - \frac{4i\epsilon}{(\alpha_1 + \alpha_2)} + \frac{\phi}{2}\right) \times \sinh\left(\frac{\tau_1 - \tau_1'}{2\alpha_1} - \frac{4i\epsilon}{(\alpha_1 + \alpha_2)} - \frac{\phi}{2}\right) \right]^{-1} \quad (30)$$

with

$$\cosh \phi = 1 + \frac{(\alpha_1 - \alpha_2)^2 + |\Delta\mathbf{x}|^2}{2\alpha_1\alpha_2} \quad (31)$$

with $|\Delta\mathbf{x}|^2 = (x_2 - x_1)^2 - (y_2 - y_1)^2$ being the orthogonal distance between the atoms (*orthogonal* here means perpendicular to the (t, z) -plane). Hence one has that:

$$F_{21}(\Delta\omega, \Delta t) = \frac{1}{2}e^{a(\xi_2-\xi_1)} \int_{-\Delta t}^{\Delta t} d\psi \int_{|\psi|+2\tau_0}^{-|\psi|+2\tau_f} d\eta \times e^{-i\Delta\omega(a_-/2\alpha_1)\eta} e^{-i\Delta\omega(a_+/2\alpha_1)\psi} G_c^+(\psi), \quad (32)$$

where we used Eq. (20) and where $a_- = \alpha_2 - \alpha_1$ and $a_+ = \alpha_2 + \alpha_1$. After some algebraic manipulations, one gets

$$\begin{aligned}
 F_{21}(\Delta\omega, \Delta t) &= \frac{-i}{16\pi^2\alpha_1\Delta\omega a_-} \\
 &\times \left\{ e^{-i\Delta\omega(a_-/\alpha_1)(\Delta t+\tau_0)} [I(\Delta\omega, \Delta t, 1) + I(\Delta\omega, \Delta t, -\alpha_2/\alpha_1)] \right. \\
 &- \left. e^{i\Delta\omega(a_-/\alpha_1)(\Delta t-\tau_f)} [I(\Delta\omega, \Delta t, -1) + I(\Delta\omega, \Delta t, \alpha_2/\alpha_1)] \right\} \quad (33)
 \end{aligned}$$

where

$$I(\Delta\omega, \Delta t, \sigma) \equiv \int_0^{\Delta t} d\psi e^{-i\sigma\Delta\omega\psi} G_{c_0}^+(\psi). \quad (34)$$

The contribution for asymptotic time interval is given by

$$\begin{aligned}
 \int_0^\infty d\psi e^{-i\sigma\Delta\omega\psi} G_{c_0}^+(\psi) &= \frac{4\alpha_1}{\sinh\phi} \sin(|\Delta\omega|\sigma\alpha_1\phi) \\
 &\times \left\{ \left[\nu_0 + \frac{\pi}{e^{2\pi\alpha_1\sigma|\Delta\omega|} - 1} + \zeta(\Delta\omega, \sigma) \right] \Theta(-\Delta\omega) \right. \\
 &+ \left. \left[\frac{\pi}{e^{2\pi\alpha_1\sigma\Delta\omega} - 1} + \zeta(\Delta\omega, \sigma) \right] \Theta(\Delta\omega) \right\}, \quad (35)
 \end{aligned}$$

where we have defined $\nu_0 = 2i \log(\alpha_1\phi) + \pi$ and the $\zeta(\Delta\omega, \sigma)$ is a combination of Hurwitz-Lerch zeta functions that is defined in the Appendix B. The expressions (33) and (35) clearly display a thermal Planck factor with a gray-body term, where the contributions with $\sigma = \pm 1$ contain information about the temperature β_1 and contributions with $\sigma = \pm \alpha_2/\alpha_1$ comprise the knowledge on the temperature β_2 . In a similar fashion, one has that

$$\begin{aligned}
 F_{12}(\Delta\omega, \Delta t) &= \frac{i}{16\pi^2\alpha_1\Delta\omega a_-} \left\{ e^{i\Delta\omega(a_-/\alpha_1)(\Delta t+\tau_0)} \right. \\
 &\times [I(\Delta\omega, \Delta t, -1) + I(\Delta\omega, \Delta t, \alpha_2/\alpha_1)] \\
 &- e^{-i\Delta\omega(a_-/\alpha_1)(\Delta t-\tau_f)} [I(\Delta\omega, \Delta t, 1) \\
 &+ I(\Delta\omega, \Delta t, -\alpha_2/\alpha_1)] \left. \right\}. \quad (36)
 \end{aligned}$$

Observe that $F_{21}(\Delta\omega, \Delta t) = F_{12}^*(\Delta\omega, \Delta t)$. Hence the object of interest is $\text{Re}[F_{12}(\Delta\omega, \Delta t)]$.

In order to study the entanglement generation, we focus attention on the particular transition $|gg\rangle \rightarrow |\Psi^+\rangle$. The corresponding matrix elements of this transition are given by

$$m_{gs}^{(1)} = m_{gs}^{(2)} = 1/\sqrt{2}, \quad (37)$$

and the gap energy is $\Delta\omega = \omega_0$. We define the cross contribution for the total transition rate as $\text{Re}[R_{12}]_{|gg\rangle \rightarrow |\Psi^+\rangle}$ which is properly evaluated in Appendix B, see the expression (B.7). The behavior of such a quantity as a function of the inverse accelerations α_1, α_2 is depicted in the Fig. 1, for a fixed small time interval and different spatial separations $|\Delta\mathbf{x}|$. One plainly observes the occurrence of maximum values for $\text{Re}[R_{12}]$ for specific values of the accelerations. This result primarily demonstrates how $|\Delta\mathbf{x}|$ can be employed as a control parameter for entanglement generation from the vacuum state. As expected, a large value of $|\Delta\mathbf{x}|$ corresponds to a significant reduction on the magnitude of the cross contribution.

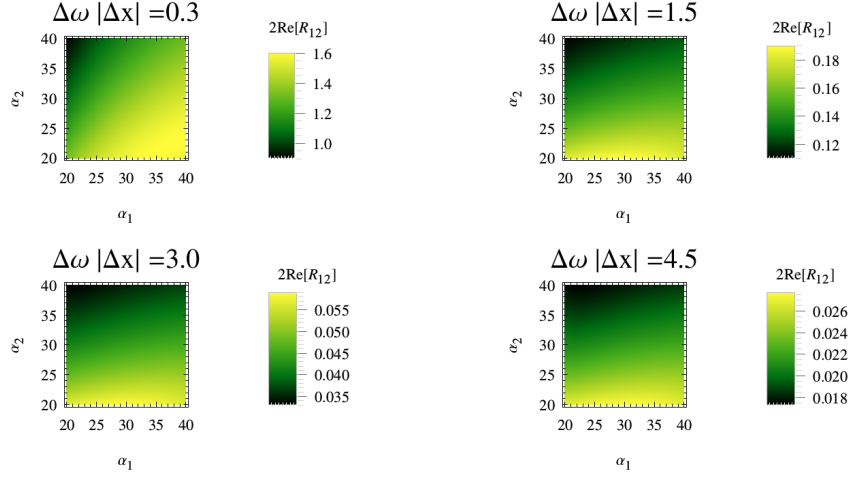


Figure 1. The quantity $\text{Re}[R_{12}]_{|gg\rangle \rightarrow |\Psi^+\rangle}$ as a function of the inverse accelerations α_1, α_2 for different values of $\Delta\omega|\Delta\mathbf{x}|$. The quantity $\Delta\omega|\Delta\mathbf{x}|$ serves as control parameter in the study of entanglement generation. All physical quantities are given in terms of the natural units associated with the specific transition $|gg\rangle \rightarrow |\Psi^+\rangle$. Therefore, in this case ξ_1, ξ_2 and α_1, α_2 are measured in units of λ , and ω_0 is given in units of $2\pi\lambda^{-1}$, where $\lambda = 2\pi/\omega_0$. Moreover, $\text{Re}[R_{12}]$ is measured in units of λ^{-1} .

The results for different time intervals are summarized in Fig. 2. Observe that the mutual influences of the atoms in this situation implies in a rather distinct interference pattern in comparison with the previous case. In addition, not only the region $\alpha_1 = \alpha_2$ in the plot produces sensible contributions to the transition rate, but we note the appearance of other regions in the plot that also provide important contributions.

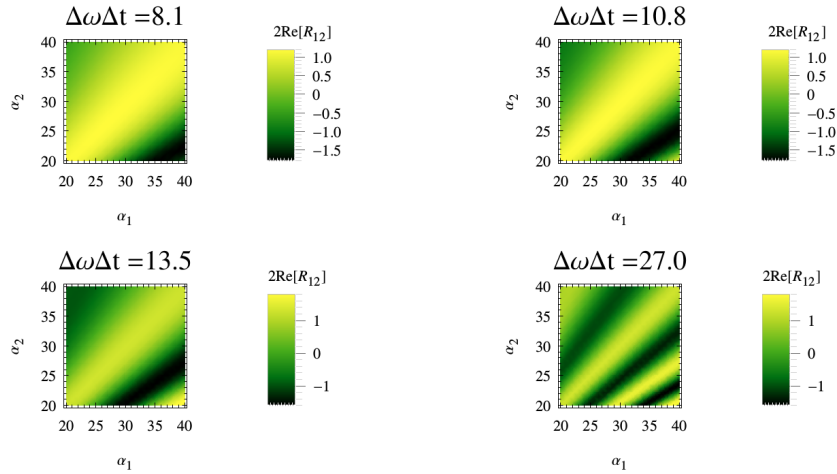


Figure 2. The quantity $\text{Re}[R_{12}]_{|gg\rangle \rightarrow |\Psi^+\rangle}$ as a function of the inverse accelerations α_1, α_2 for different values of $\Delta\omega\Delta t$. We consider a fixed value $\Delta\omega|\Delta\mathbf{x}| = 0.3$. Here it is clear that maximum values show up in other regions besides the region in which $\alpha_1 = \alpha_2$. The inverse accelerations α_1, α_2 are measured in units of λ and $\text{Re}[R_{12}]$ is measured in units of λ^{-1} (see caption of Fig.1 for a definition of λ).

In virtue of the discussion just exposed, let us examine more closely the case in which $\alpha_1 = \alpha_2 = \alpha$. In this case, $G_c^+(\psi)$ becomes $G_\alpha^+(\psi)$ which is defined as

$$G_\alpha^+(\psi) = \frac{-(4\sqrt{2}\pi\alpha)^{-2}}{\sinh\left(\frac{\psi}{2\alpha} - \frac{2i\epsilon}{\alpha} + \frac{\phi}{2}\right) \sinh\left(\frac{\psi}{2\alpha} - \frac{2i\epsilon}{\alpha} - \frac{\phi}{2}\right)}. \quad (38)$$

The expression (32) then reads,

$$F_{21}(\Delta\omega, \Delta t) = \int_{-\Delta t}^{\Delta t} d\psi \int_{|\psi|+2\tau_0}^{-|\psi|+2\tau_f} d\eta e^{-i\Delta\omega\psi} G_\alpha^+(\psi). \quad (39)$$

One can resort to contour integration methods in order to evaluate the integral in ψ . Observe that the integrand have simple poles given by

$$\psi = 2\pi\alpha in + 4i\epsilon \pm \alpha\phi, \quad (40)$$

where n is an integer. For $\Delta\omega < 0$ we make use of an infinite semicircle that we close on the upper-half $\text{Im}[\psi] > 0$ plane; for $n \neq 0$ one may take the limit $\epsilon \rightarrow 0$ before solving the integral. This contour encloses the poles for $n \geq 0$ and runs in an anticlockwise direction. For $\Delta\omega > 0$ we close the contour in an infinite semicircle in the lower-half $\text{Im}[\psi] < 0$ plane. Now, this contour encloses the poles for $n < 0$ and runs in the clockwise direction (see Fig. A1). In the asymptotic limit, we have

$$\begin{aligned} \lim_{\Delta t \rightarrow \infty} R_{21}(\Delta\omega, \Delta t) &= \frac{\sin(|\Delta\omega|\alpha\phi)}{2\pi\alpha \sinh \phi} \left\{ \Theta(-\Delta\omega) \left[1 + \frac{1}{e^{2\pi\alpha|\Delta\omega|} - 1} \right] \right. \\ &\quad \left. + \Theta(\Delta\omega) \frac{1}{e^{2\pi\alpha\Delta\omega} - 1} \right\}. \end{aligned} \quad (41)$$

In a similar way, we obtain the same asymptotic limit for $R_{12}(\Delta\omega, \Delta t)$. For the specific transition $|gg\rangle \rightarrow |\Psi^+\rangle$, with matrix elements given by Eq. (37), one has that

$$\text{Re}[R_{12}(\omega_0)]_{|gg\rangle \rightarrow |\Psi^+\rangle} = \text{Re} \left[\frac{\sin(\omega_0\alpha\phi)}{2\pi\alpha \sinh \phi} \frac{1}{e^{2\pi\alpha\omega_0} - 1} \right]. \quad (42)$$

To conclude this Section, let us discuss the total transition rate (17) within the asymptotic time interval regime and for small distances between the atoms. We also consider the case of equal accelerations, in which the cross contributions are given by the Eq. (42). In this situation one can express the total transition rate as follows

$$\mathcal{R}_{|gg\rangle \rightarrow |\Psi^+\rangle} = R_{11}f(\omega_0\alpha\phi), \quad (43)$$

where $f(x) = 2(1 + \sin x/x)$. This function quantifies the influence of the crossed response functions on the entanglement between atoms, for asymptotic time intervals. Some special values are given by (n is a positive integer)

$$f[(2n+1)\pi/2] = 2 \left(1 + \frac{2(-1)^n}{(2n+1)\pi} \right). \quad (44)$$

Let us discuss the behaviour of this function more closely. There is a great oscillatory regime for large accelerations and small distances between the atoms. Since the atoms have the same z coordinate, for $\phi \ll 1/\Delta\omega\alpha$ cross correlations are more important for the rate (43) in comparison with cases in which the $\Delta\omega|\Delta\mathbf{x}|$ becomes larger. Therefore, the

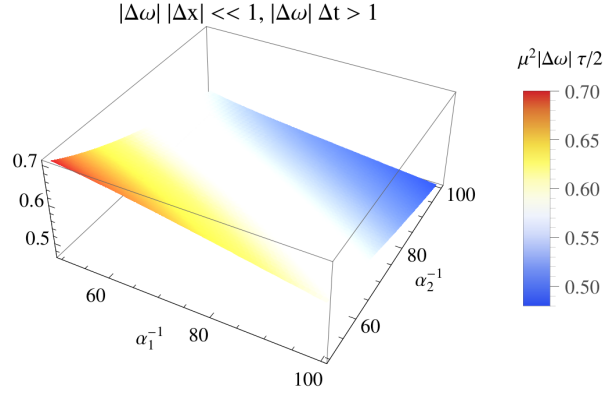


Figure 3. The quantity $\mu^2|\Delta\omega|\tau/2$ as a function of the accelerations of the atoms. We consider the fixed values $|\Delta\omega||\Delta\mathbf{x}| = 0.3$ and $|\Delta\omega|\Delta t = 1.2$. The inverse accelerations α_1, α_2 are measured in units of λ (see caption of Fig.1 for a definition of λ).

crossed response functions generate a constructive interference when the atoms are near each other in space. In turn, these interference terms vanish for large spatial separations between the atoms. Similar conclusions were reported in Refs. [11, 20, 21, 30, 31].

4. Mean life of entangled states

So far we have studied the formation of entangled states through the excitation of the collective ground state $|gg\rangle$. We have demonstrated that, for uniformly accelerated atoms, the interaction with a common quantum field can act as a source of entanglement. On the other hand, such an interaction can also induce decoherence effects. Hence a natural question that emerges is whether such entangled states persist for long time intervals. A possible measurement of the decay of entangled states is given by the mean life of such states. The mean life for the state $|\omega'\rangle$ is defined as

$$\tau_{|\omega'\rangle \rightarrow |\omega\rangle}(\Delta\omega, \Delta t) = [\mathcal{R}_{|\omega'\rangle \rightarrow |\omega\rangle}(\Delta\omega, \Delta t)]^{-1}. \quad (45)$$

In order to study the stability of the entangled states under spontaneous emission processes one can study the related transition $|\Psi^+\rangle \rightarrow |gg\rangle$. The corresponding matrix elements of this transition are again given by Eq. (37), and the gap energy now is given by $\Delta\omega = -\omega_0$. Hence the expression (45) becomes

$$\begin{aligned} \tau_{|\Psi^+\rangle \rightarrow |gg\rangle} &= \frac{2}{\mu^2} \left\{ R_{11}(-\omega_0, \Delta t) + R_{22}(-\omega_0, \Delta t) \right. \\ &\quad \left. + 2\text{Re}[R_{12}(-\omega_0, \Delta t)] \right\}^{-1}. \end{aligned} \quad (46)$$

The behavior of the mean life as a function of the accelerations for a relatively small time interval and with the condition $|\Delta\omega||\Delta\mathbf{x}| \ll 1$ is depicted in the Fig. 3. Note that such a function falls off quickly with the acceleration. This result has a clear-cut meaning: quantum entanglement disappears for sufficiently large accelerations. Fig. 4 presents a similar situation as the previous figure but with $|\Delta\omega|\Delta t \gg 1$. We note the

emergence of an oscillatory regime. This implies that the mean life of the entangled states displays maximum values at given accelerations of the atoms. In such a scenario one concludes that the mutual influence of atoms will contribute to the entanglement stability only for long times intervals. On the other hand, Fig. 5 shows that, for larger values of $|\Delta\omega||\Delta\mathbf{x}|$, the oscillations are less severe than the previous case.

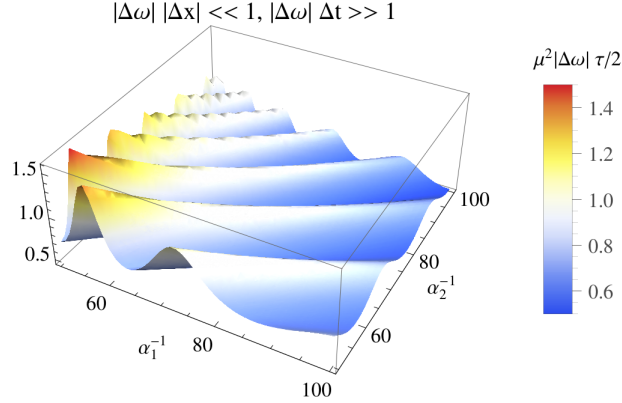


Figure 4. The quantity $\mu^2|\Delta\omega|\tau/2$ as a function of the accelerations of the atoms. We consider the fixed values $|\Delta\omega||\Delta\mathbf{x}| = 0.3$ and $|\Delta\omega|\Delta t = 60.0$. We have the presence of interference effects that provide more stability for the entangled states. The inverse accelerations α_1, α_2 are measured in units of λ (see caption of Fig.1 for a definition of λ).

5. Summary and outlook

In this paper we have studied two identical uniformly accelerated two-level atoms weakly coupled with a massless scalar field prepared in the Minkowski vacuum state. We have shown the possibility of generation of entanglement between such two atoms initially prepared in the ground state. We also found that the associated response function contains terms related to cross correlations between the atoms mediated by the field. Since the atoms move along different world lines, such crossed terms present thermal contributions with different temperatures. The crossed terms of the response function are modulated by an oscillating function. In addition, such contributions are accompanied by a gray body factor, composed by a linear combination of derivatives of Hurwitz-Lerch zeta-functions, whose arguments have information on the accelerations and the energy gap. The appearance of the aforementioned gray body factor may be understood in the sense of Ref. [10], in which the authors demonstrate the emergence of a thermal noise produced by the fluctuations of the fields and field correlations between the two trajectories. Moreover, we also presented a general expression for the mean life of the entangled states. In general, we found that atoms with same acceleration will be less correlated for an increasing $|\Delta\mathbf{x}|$, as expected.

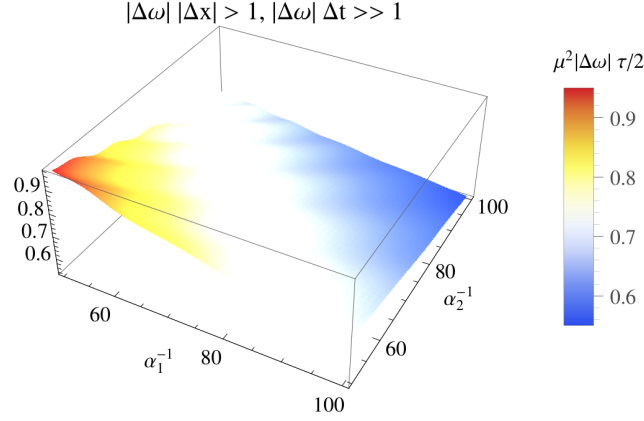


Figure 5. The quantity $\mu^2|\Delta\omega|\tau/2$ as a function of the accelerations of the atoms. We consider the fixed values $|\Delta\omega||\Delta\mathbf{x}| = 3$ and $|\Delta\omega|\Delta t = 60.0$. The inverse accelerations α_1, α_2 are measured in units of λ (see caption of Fig.1 for a definition of λ).

Acknowledgements

This work was partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico – CNPq and Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – CAPES (Brazilian agencies).

Appendix A. Explicit calculation of F_{11} and F_{22}

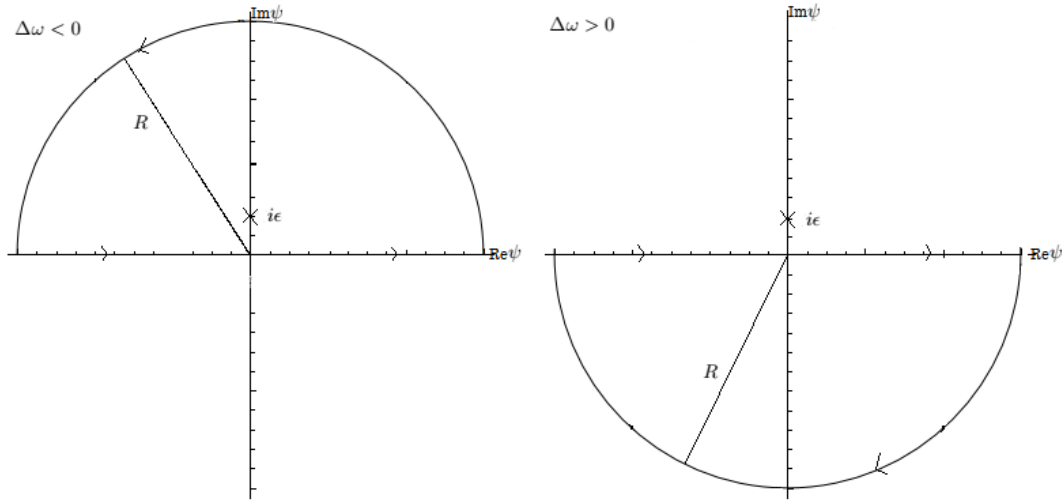


Figure A1. Contour used to perform the integral of $R_{11}(\Delta\omega, \Delta t)$ and $R_{22}(\Delta\omega, \Delta t)$.

In this Appendix we concisely perform the evaluation of the individual contributions of the atoms to the total response function. In order to study the contribution (21), we perform the Fourier transform with the help of contour integration methods. From the

expression (19) one notes the existence of second order poles of the form

$$\psi = 2i\epsilon - 2\pi i\alpha_1 n, \quad (\text{A.1})$$

where n is an integer. One must treat separately the cases of $n \neq 0$ and $n = 0$. For $n \neq 0$ we may take the limit $\epsilon \rightarrow 0$ before solving the integral. For $\Delta\omega < 0$ we make use of a semicircle of radius R that we close on the upper-half $\text{Im}[\psi] > 0$ plane. This contour encloses the poles for $n \geq 0$ and runs in an anticlockwise direction. For $\Delta\omega > 0$ we close the contour in a semicircle of radius R in the lower-half $\text{Im}[\psi] < 0$ plane. Now, this contour encloses the poles for $n < 0$ and runs in the clockwise direction (see Fig. A1). We consider the limit $R \rightarrow \infty$ such that the contribution from the arcs will vanish by the Jordan's lemma. We obtain, for the atom 1

$$\begin{aligned} F_{11}(\Delta\omega, \Delta t) = & \frac{\Delta t}{2\pi^2} \left\{ \pi|\Delta\omega|\Theta(-\Delta\omega) + |\Delta\omega| \left(\text{Si}(\Delta\omega\Delta t) - \frac{\pi}{2} \right) \right. \\ & + \frac{\pi|\Delta\omega|}{e^{2\pi\alpha_1|\Delta\omega|} - 1} \\ & + \int_{\Delta t}^{\infty} d\psi \cos(\Delta\omega\psi) \left[\frac{1/(2\alpha_1)^2}{\sinh^2[\psi/(2\alpha_1)]} - \frac{1}{\psi^2} \right] \Big\} \\ & + \frac{1}{2\pi^2} \left\{ \cos(\Delta\omega\Delta t) + \log\left(\frac{\Delta t}{2\pi\epsilon}\right) - 1 \right. \\ & + \int_0^{\Delta t} d\psi \frac{\cos(\Delta\omega\psi) - 1}{\psi} \\ & + \left. \int_0^{\Delta t} d\psi \psi \cos(\Delta\omega\psi) \left[\frac{1/(2\alpha_1)^2}{\sinh^2[\psi/(2\alpha_1)]} - \frac{1}{\psi^2} \right] \right\}. \quad (\text{A.2}) \end{aligned}$$

This is the expression (22). By the definition (23), we have

$$\begin{aligned} R_{11}(\Delta\omega, \Delta t) = & \frac{|\Delta\omega|}{2\pi} \left\{ \Theta(-\Delta\omega) + \frac{1}{e^{2\pi\alpha_1|\Delta\omega|} - 1} \right. \\ & + \frac{\cos(\Delta\omega\Delta t)}{\pi\Delta\omega\Delta t} + \frac{\text{Si}(\Delta\omega\Delta t)}{\pi} - \frac{1}{2} \Big\} \\ & + \frac{1}{2\pi^2} \int_{\Delta t}^{\infty} d\psi \cos(\Delta\omega\psi) \left[\frac{1/(2\alpha_1)^2}{\sinh^2[\psi/(2\alpha_1)]} - \frac{1}{\psi^2} \right], \quad (\text{A.3}) \end{aligned}$$

where $\text{Si}(z)$ is the sine integral function given by [32]

$$\text{Si}(z) = \int_0^z \frac{\sin t}{t} dt. \quad (\text{A.4})$$

In an analogous way, for the individual contribution of the atom 2 given by Eq. (27) one has that (using the same contour depicted in the Fig. (A1))

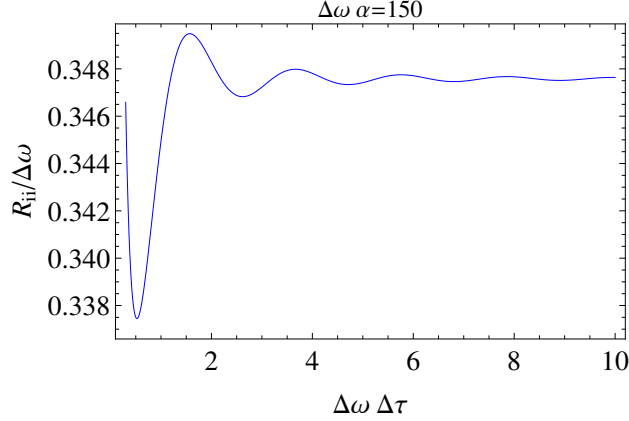


Figure A2. Behaviour of the individual contributions $R_{ii}(\Delta\omega)$ for $\Delta\omega\alpha = 150$. The transition probability increases with the acceleration.

$$\begin{aligned}
 F_{22}(\Delta\omega, \Delta t) = & \frac{\Delta t}{2\pi^2} \left\{ \pi|\Delta\omega|\Theta(-\Delta\omega) + |\Delta\omega| \left(\text{Si}(\Delta\omega\Delta t) - \frac{\pi}{2} \right) \right. \\
 & + \frac{\pi|\Delta\omega|}{e^{2\pi\alpha_2|\Delta\omega|} - 1} \\
 & + \int_{\Delta t}^{\infty} d\psi \cos(\Delta\omega\psi) \left[\frac{1/(2\alpha_2)^2}{\sinh^2[\psi/(2\alpha_2)]} - \frac{1}{\psi^2} \right] \Big\} \\
 & + \frac{1}{2\pi^2} \left\{ \cos(\Delta\omega\Delta t) + \log\left(\frac{\Delta t}{2\pi\epsilon}\right) - 1 \right. \\
 & + \int_0^{\Delta t} d\psi \frac{\cos(\Delta\omega\psi) - 1}{\psi} \\
 & + \left. \int_0^{\Delta t} d\psi \psi \cos(\Delta\omega\psi) \left[\frac{1/(2\alpha_2)^2}{\sinh^2[\psi/(2\alpha_2)]} - \frac{1}{\psi^2} \right] \right\}, \quad (\text{A.5})
 \end{aligned}$$

and consequently

$$\begin{aligned}
 R_{22}(\Delta\omega, \Delta t) = & \frac{|\Delta\omega|}{2\pi} \left\{ \Theta(-\Delta\omega) + \frac{1}{e^{2\pi\alpha_2|\Delta\omega|} - 1} \right. \\
 & + \frac{\cos(\Delta\omega\Delta t)}{\pi\Delta\omega\Delta t} + \frac{\text{Si}(\Delta\omega\Delta t)}{\pi} - \frac{1}{2} \Big\} \\
 & + \frac{1}{2\pi^2} \int_{\Delta t}^{\infty} d\psi \cos(\Delta\omega\psi) \left[\frac{1/(2\alpha_2)^2}{\sinh^2[\psi/(2\alpha_2)]} - \frac{1}{\psi^2} \right]. \quad (\text{A.6})
 \end{aligned}$$

Appendix B. Explicit calculation of F_{12} and F_{21}

In this Appendix we perform the explicit evaluation of the cross contributions F_{12} and F_{21} . As above, we shall employ the method of residues. The integral (34) can be expressed as,

$$I(\Delta\omega, \Delta t, \sigma) = \int_0^{\infty} d\psi e^{-i\sigma\Delta\omega\psi} G_{c_0}^+(\psi) - \int_{\Delta t}^{\infty} d\psi e^{-i\sigma\Delta\omega\psi} G_{c_0}^+(\psi). \quad (\text{B.1})$$

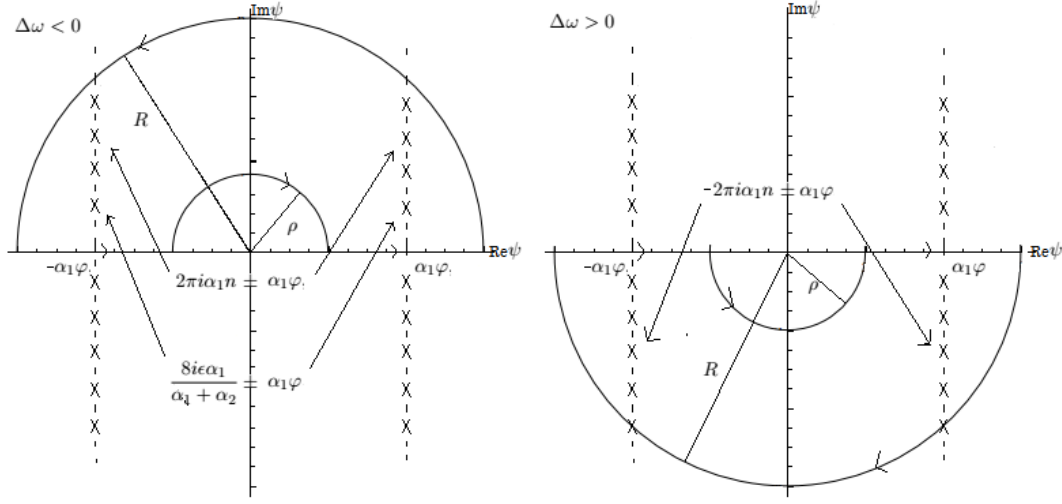


Figure B1. Contour used for perform the integral of $R_{12}(\Delta\omega, \Delta t)$ and $R_{21}(\Delta\omega, \Delta t)$.

For the first term on the right-hand side of the expression (B.1), the simple poles of the integrand are given by

$$\psi_n = 2\pi i \alpha_1 n + \frac{8i\epsilon\alpha_1}{\alpha_1 + \alpha_2} \pm \alpha_1 \phi, \quad (\text{B.2})$$

where n is an integer. Making use of the following auxiliary integrals

$$\oint_C dz e^{-i\Delta\omega\sigma z} \log(z) G_{c_0}^+(z), \quad \oint_C dz e^{-i\Delta\omega\sigma z} G_{c_0}^+(z),$$

we shall perform the integral for $\Delta\omega < 0$ with a contour such that it encloses the poles for $n \geq 0$ and runs in an anticlockwise direction. For processes with $\Delta\omega > 0$ we employ a contour such that it encloses the poles for $n < 0$ and runs in the clockwise direction (in this case one may perform the limit $\epsilon \rightarrow 0$ before the evaluation of integral, see Fig.B1). In the limit $\rho \rightarrow 0$ and $R \rightarrow \infty$ and we obtain the following expression

$$\begin{aligned} \int_0^\infty d\psi e^{-i\sigma\Delta\omega\psi} G_{c_0}^+(\psi) &= \frac{4\alpha_1}{\sinh\phi} \sin(|\Delta\omega|\sigma\alpha_1\phi) \\ &\times \left\{ \left[\nu_0 + \frac{\pi}{e^{2\pi\alpha_1\sigma|\Delta\omega|} - 1} + \zeta(\Delta\omega, \sigma) \right] \Theta(-\Delta\omega) \right. \\ &\left. + \left[\frac{\pi}{e^{2\pi\alpha_1\sigma\Delta\omega} - 1} + \zeta(\Delta\omega, \sigma) \right] \Theta(\Delta\omega) \right\}. \end{aligned} \quad (\text{B.3})$$

This is the expression (35), where we have defined the function

$$\zeta(\Delta\omega, \sigma) = \zeta_1(\Delta\omega, \sigma) + \cot(|\Delta\omega|\alpha_1\sigma\phi)\zeta_2(\Delta\omega, \sigma), \quad (\text{B.4})$$

with

$$\begin{aligned} \zeta_1(\Delta\omega, \sigma) &= -ie^{-2\pi\alpha_1\sigma\Delta\omega} \left\{ \text{Re} \left(\Phi^{(0,1,0)}(e^{-2\pi\alpha_1\sigma\Delta\omega}, 0, \chi) \right) \right. \\ &\quad + e^{2\pi\alpha_1\sigma\Delta\omega} \left[1 + \log(2\alpha_1\sqrt{\pi}) \right] \\ &\quad \left. + e^{2\pi\alpha_1\sigma\Delta\omega} \text{Re} \left(\Phi^{(0,1,0)}(e^{2\pi\alpha_1\sigma\Delta\omega}, 0, \chi) \right) \right\} \end{aligned} \quad (\text{B.5})$$

and

$$\begin{aligned} \zeta_2(\Delta\omega, \sigma) = & 8ie^{-2\pi\alpha_1\sigma\Delta\omega} \left\{ \text{Im} \left(\Phi^{(0,1,0)}(e^{-2\pi\alpha_1\sigma\Delta\omega}, 0, \chi) \right) \right. \\ & \left. - e^{2\pi\alpha_1\sigma\Delta\omega} \left[\frac{\pi}{2} + e^{2\pi\alpha_1\sigma\Delta\omega} \text{Im} \left(\Phi^{(0,1,0)}(e^{2\pi\alpha_1\sigma\Delta\omega}, 0, \chi) \right) \right] \right\}. \end{aligned} \quad (\text{B.6})$$

In the above, $\Phi(z, s, \alpha)$ the Hurwitz–Lerch zeta-function, $\Phi^{(0,1,0)}(z, s, \alpha)$ is its first derivative with respect to its second variable and $\chi = 1 + (i\phi/2\pi)$. Recalling Eq. (33), we observe that the contributions with $\sigma = 1$ are associated with $\beta_1^{-1} = 1/2\pi\alpha_1$ whereas the contributions with $\sigma = \alpha_2/\alpha_1$ are related to $\beta_2^{-1} = 1/2\pi\alpha_2$. The contribution of the cross correlations to the total transition rate is given by the real part of the following expression

$$\begin{aligned} [R_{12}]_{|gg\rangle \rightarrow |\Psi^+\rangle} = & \frac{i}{16\pi^2\alpha_1^2\omega_0a_-} \\ & \times \left\{ i\omega_0a_- \left[e^{i\omega_0(a_-/\alpha_1)(\Delta t + \tau_0)} [I(\omega_0, \Delta t, -1) + I(\omega_0, \Delta t, \alpha_2/\alpha_1)] \right. \right. \\ & + e^{-i\omega_0(a_-/\alpha_1)(\Delta t - \tau_f)} [I(\omega_0, \Delta t, 1) + I(\omega_0, \Delta t, -\alpha_2/\alpha_1)] \left. \right] \\ & + \alpha_1 G_{c_0}^+(\Delta t) \left[e^{i\omega_0(a_-/\alpha_1)\tau_0} (e^{i\omega_0\Delta t\alpha_2/\alpha_1} + e^{i\omega_0\Delta t(2\alpha_2 - \alpha_1)/\alpha_1}) \right. \\ & \left. \left. - e^{i\omega_0(a_-/\alpha_1)\tau_f} (e^{-i\omega_0\Delta t\alpha_2/\alpha_1} + e^{-i\omega_0\Delta t(2\alpha_2 - \alpha_1)/\alpha_1}) \right] \right\}. \end{aligned} \quad (\text{B.7})$$

References

- [1] Davies P C W 1975 *J. Phys. A* **8** 609
- [2] Unruh W G 1976 *Phys. Rev. D* **14** 870
- [3] DeWitt B S 1975 *Phys. Rep.* **19** 295
- [4] Sciamia D W, Candelas P and Deutsch D 1981 *Adv. Phys.* **30** 327
- [5] Crispino L C B, Higuchi A and Matsas G E A 2008 *Rev. Mod. Phys.* **80** 787
- [6] Bouwmeester D, Ekert A K and Zeilinger A (eds) 2000 *The Physics of Quantum Information* (Berlin: Springer-Verlag)
- [7] Plenio M B, Huelga S F, Beige A and Knight P L 1999 *Phys. Rev. A* **59** 2468
- [8] Basharov A M 2002 *Jetp Lett.* **75** 123
- [9] Carvalho A R R, Busse M, Brodier O, Viviescas C and Buchleitner A 2007 *Phys. Rev. Lett.* **98** 190501
- [10] Raval A, Hu B L and Anglin J 1996 *Phys. Rev. D* **53** 7003
- [11] Lin S-Y and Hu B L 2009 *Phys. Rev. D* **79** 085020
- [12] Lin S-Y and Hu B L 2010 *Phys. Rev. D* **81** 045019
- [13] and Mann R B 2005 *Phys. Rev. Lett.* **95** 120404
- [14] Bal J L, Fuentes-Schuller I and Schuller F P 2006 *Phys. Lett. A* **359** 550
- [15] Cliche M and Kempf A 2010 *Phys. Rev. A* **81** 012330
- [16] Reznik B 2003 *Found. Phys.* **33** 167
- [17] Cliche M and Kempf A 2011 *Phys. Rev. D* **83** 045019
- [18] Martín-Martínez E, Smith A R H and Terno D R 2016 *Phys. Rev. D* **93** 044001
- [19] Rideout D *et al* 2012 *Class. Quant. Grav.* **29** 224011
- [20] Menezes G and Svaiter N F 2015 *Phys. Rev. A* **92** 062131
- [21] Menezes G and Svaiter N F 2016 *Phys. Rev. A* **93** 052117
- [22] Menezes G 2016 *Phys. Rev. D* **94** 105008
- [23] Menezes G 2017 *Phys. Rev. D* (to be published) (*Preprint* gr-qc/1611.00056)
- [24] Svaiter B F and Svaiter N F 1992 *Phys. Rev. D* **46** 5267
- [25] Ford L H, Svaiter N F and Lyra M L 1994 *Phys. Rev. A* **49** 1378

- [26] Sriramkumar L and Padmanabhan T 1996 *Class. Quant. Grav.* **13** 2061
- [27] Birrell N D and Davies P C W 1984 *Quantum Fields in Curved Space* (Cambridge: Cambridge University Press)
- [28] Dicke R H 1954 *Phys. Rev.* **93** 99
- [29] Gradshteyn I S and Ryzhik I M 2007 *Table of Integrals, Series, and Products* (Amsterdam: Elsevier Academic Press)
- [30] Lenz G and Meystre P 1993 *Phys. Rev. A* **48** 3365
- [31] Arias E, Dueñas J G, Menezes G and Svaiter N F 2016 *JHEP* **2016** 147
- [32] Abramowitz M and Stegun I A 1965 *Handbook of Mathematical Functions: with Formulas, Graphs, and Mathematical Tables* (New York: Dover Publications)